

# Extra Dirac Equations\*

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## Abstract

This paper has rather a pedagogical meaning. Surprising symmetries in the  $(j, 0) \oplus (0, j)$  Lorentz group representation space are analyzed. The aim is to draw reader's attention to the possibility of describing the particle world on the ground of the Dirac "doubles". Several tune points of the variational principle for this kind of equations are briefly discussed.

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Chiral invariance has profound significance in the modern theory of weak interactions and it has played a prominent role in understanding the low energy properties of strong interactions. But, it has long been recognized that the Dirac Lagrangian for massive fermions is invariant only under simultaneous chiral transformation  $\psi \rightarrow \gamma_5 \psi$  and “mass reversal” transformation  $m \rightarrow -m$ . The idea of construction of particle dynamics on the ground of two Dirac equations (the second one has the opposite sign at the mass term) has been proposed shortly after an appearance of the famous equation [1,2].<sup>1</sup> Unfortunately, both those investigations and the papers [3] have been forgotten. Another possibility of two extra Dirac equations (two extra “square roots” of the Klein-Gordon equation) for 4-spinors of the second kind [4] seems also to have escaped from attention of theoreticians. The only mention of this possibility, that I was able to find, is in ref. [5, Eq.(8)], in connection with an “anomalous” representation of the inversion group [6]. Some speculations on the possible relevance of such the kind of equations to description of neutrino and on the eventual connection with existence of isotopic spin have been presented there. In this essay I am going to undertake a detailed analysis of the Dirac “doubles”<sup>2</sup> and to found some relations with the models discussed recently [7–10].

Beforehand, I would like to reproduce here the way of deriving the usual Dirac equation on the ground of the Wigner rules [11] of Lorentz transformations of the  $(0, j)$  left  $\phi_L(p^\mu)$  and the  $(j, 0)$  right  $\phi_R(p^\mu)$  spinors:

$$(j, 0) : \quad \phi_R(p^\mu) = \Lambda_R(p^\mu \leftarrow \hat{p}^\mu) \phi_R(\hat{p}^\mu) = \exp(+\mathbf{J} \cdot \boldsymbol{\varphi}) \phi_R(\hat{p}^\mu) \quad , \quad (1a)$$

$$(0, j) : \quad \phi_L(p^\mu) = \Lambda_L(p^\mu \leftarrow \hat{p}^\mu) \phi_L(\hat{p}^\mu) = \exp(-\mathbf{J} \cdot \boldsymbol{\varphi}) \phi_L(\hat{p}^\mu) \quad . \quad (1b)$$

It is known and is given, *e.g.*, in the papers [12, p.9], [13, p.43-44]. In ref. [10b, footnote # 1] several important points have been shown at: “Refer to Eqs. (1a) and (1b) and set  $\mathbf{J} = \boldsymbol{\sigma}/2$ ... Spinors [implied by the arguments based on parity symmetry and that Lorentz group is essentially  $SU_R(2) \otimes SU_L(2)$ ] turn out to be of crucial significance in constructing a field  $\Psi(x)$  that describes eigenstates of the Charge operator,  $Q$ , if [in the rest]

$$\phi_R(\hat{p}^\mu) = \pm \phi_L(\hat{p}^\mu) \quad (2)$$

(otherwise physical eigenstates are no longer charge eigenstates). We call [this relation], the “Ryder-Burgard relation”... Next couple the Ryder-Burgard relation with Eqs. (1a) and (1b) to obtain

$$\begin{pmatrix} \mp m \mathbb{1} & p_0 + \boldsymbol{\sigma} \cdot \mathbf{p} \\ p_0 - \boldsymbol{\sigma} \cdot \mathbf{p} & \mp m \mathbb{1} \end{pmatrix} \psi(p^\mu) = 0 \quad . \quad (3)$$

[Above we have used the property  $[\Lambda_{L,R}(p^\mu \leftarrow \hat{p}^\mu)]^{-1} = [\Lambda_{R,L}(p^\mu \leftarrow \hat{p}^\mu)]^\dagger$  and that both  $\mathbf{J}$  and  $\Lambda_{R,L}$  are Hermitian for the finite  $(j = 1/2, 0) \oplus (0, j = 1/2)$  representation of the Lorentz

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<sup>1</sup>More exactly, in the papers of 1937 Prof. M. A. Markov considered the second-order equation for 4-component spinor. But, in the latest work he has shown that it is equivalent to the first order equation for the eight-component function (or to the set of two Dirac equations).

<sup>2</sup>The meaning of this terminology will become clear in the sequel.

group.] Introducing  $\psi(x) \equiv \psi(p^\mu) \exp(\mp i p \cdot x)$  and letting  $p_\mu \rightarrow i\partial_\mu$ , the above equation becomes:  $(i\gamma^\mu \partial_\mu - m \mathbb{1})\psi(x) = 0$ . This is the Dirac equation for spin-1/2 particles with  $\gamma^\mu$  in the Weil/Chiral representation.” In the standard (generalized canonical) representation such a choice of the Faustov-Ryder-Burgard relation<sup>3</sup> and the use of a representation of the  $\mathbf{J}$  matrices in which  $J_z$  is diagonal imply the well-known spinorial basis of bispinors  $u_h$  and  $v_h$  in the  $(j, 0) \oplus (0, j)$  representation space [7]:

$$u_{+j}(\overset{\circ}{p}^\mu) = \begin{pmatrix} N(j) \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix}, \quad u_{j-1}(\overset{\circ}{p}^\mu) = \begin{pmatrix} 0 \\ N(j) \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix}, \quad \dots, \quad v_{-j}(\overset{\circ}{p}^\mu) = \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ N(j) \end{pmatrix}. \quad (4)$$

The normalization factor is convenient to choose  $N(j) = m^j$  in order the rest spinors to vanish in the massless limit.

The pioneer study of the  $(j, 0) \oplus (0, j)$  representation space has been undertaken by Weinberg [14] in the sixties<sup>4</sup>. The use of the Faustov-Ryder-Burgard relation (2) in the case of  $j = 1$  permits us to obtain the Weinberg equation; exactly, its modified version [7]:

$$(\gamma^{\mu\nu} \partial_\mu \partial_\nu + \wp_{u,v} m^2) \psi(x) = 0 \quad (5)$$

with  $\wp_{u,v} = \pm 1$  and  $\gamma_{\mu\nu}$  are the Barut-Muzinich-Williams  $j = 1$  matrices [15]. A boson described by Eq. (5) has the opposite relative intrinsic parity with respect to its antiboson. This is an example of the Bargmann-Wightman-Wigner (BWW) type quantum field theory [16].<sup>5</sup> An explicit construct of this case of the BWW theories has been proposed recently [7] by Ahluwalia (see also ref. [20]) and has been analyzed in ref. [8].

Now, let me utilize the Faustov-Ryder-Burgard relation in a slightly different form. Namely, let assume that right- and left- complex-valued spinors are connected in the rest frame in the following way:

$$\phi_R(\overset{\circ}{p}^\mu) = \pm i \phi_L(\overset{\circ}{p}^\mu) \quad . \quad (6)$$

In fact, this choice (6) corresponds to the following spinorial basis of bispinors<sup>6</sup> (provided that 2-spinors are chosen as  $\phi_R^+(\overset{\circ}{p}^\mu) = \text{column}(1 \ 0)$  and  $\phi_R^-(\overset{\circ}{p}^\mu) = \text{column}(0 \ 1)$ ):

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<sup>3</sup>More general form of the relation (2) has been given in the unpublished preprint of Prof. Faustov [12, Eq.(22a)].

<sup>4</sup>Of course, the  $j = 1/2$  Dirac fermions are also contained in this scheme.

<sup>5</sup>About connections of this type of Poincaré invariant theories with the constructs proposed by Foldy and Nigam [17] and by Gelfand, Tsetlin and Sokolik [6,5] even before an appearance ref. [16] see ref. [10,18,19].

<sup>6</sup>Here and below I work in the chiral representation of  $\gamma$  matrices.

$$\Upsilon_+(\hat{p}^\mu) = \sqrt{\frac{m}{2}} \begin{pmatrix} 1 \\ 0 \\ -i \\ 0 \end{pmatrix} , \quad \Upsilon_-(\hat{p}^\mu) = \sqrt{\frac{m}{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -i \end{pmatrix} , \quad (7a)$$

$$\mathcal{B}_+(\hat{p}^\mu) = \sqrt{\frac{m}{2}} \begin{pmatrix} 1 \\ 0 \\ i \\ 0 \end{pmatrix} , \quad \mathcal{B}_-(\hat{p}^\mu) = \sqrt{\frac{m}{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ i \end{pmatrix} . \quad (7b)$$

If couple Eq. (6) with (1a) and (1b) one obtains the equations in the momentum representation:<sup>7</sup>

$$[i\gamma^5 \hat{p} - m] \Upsilon_{+,-}(p^\mu) = 0 , \quad (8a)$$

$$[i\gamma^5 \hat{p} + m] \mathcal{B}_{+,-}(p^\mu) = 0 , \quad (8b)$$

satisfied by the 4-spinors of the second kind:

$$\Upsilon_+(p^\mu) = \frac{1}{2\sqrt{p_0+m}} \begin{pmatrix} p^+ + m \\ p_r \\ -i(p^- + m) \\ i p_r \end{pmatrix} = \gamma^5 \mathcal{B}_+(p^\mu) = \gamma^5 S_{[1/2]}^c \Upsilon_-(p^\mu) , \quad (9a)$$

$$\Upsilon_-(p^\mu) = \frac{1}{2\sqrt{p_0+m}} \begin{pmatrix} p_l \\ p^- + m \\ i p_l \\ -i(p^+ + m) \end{pmatrix} = \gamma^5 \mathcal{B}_-(p^\mu) = -\gamma^5 S_{[1/2]}^c \Upsilon_+(p^\mu) , \quad (9b)$$

$$\mathcal{B}_+(p^\mu) = \frac{1}{2\sqrt{p_0+m}} \begin{pmatrix} p^+ + m \\ p_r \\ i(p^- + m) \\ -i p_r \end{pmatrix} = \gamma^5 \Upsilon_+(p^\mu) = -\gamma^5 S_{[1/2]}^c \mathcal{B}_-(p^\mu) , \quad (9c)$$

$$\mathcal{B}_-(p^\mu) = \frac{1}{2\sqrt{p_0+m}} \begin{pmatrix} p_l \\ p^- + m \\ -i p_l \\ i(p^+ + m) \end{pmatrix} = \gamma^5 \Upsilon_-(p^\mu) = \gamma^5 S_{[1/2]}^c \mathcal{B}_+(p^\mu) . \quad (9d)$$

We have used above the following notation:  $p_r = p_x + ip_y$ ,  $p_l = p_x - ip_y$ ,  $p^\pm = p_0 \pm p_z$  and

$$S_{[1/2]}^c = \begin{pmatrix} 0 & i\Theta_{[1/2]} \\ -i\Theta_{[1/2]} & 0 \end{pmatrix} \mathcal{K} , \quad (10)$$

with  $\mathcal{K}$  being the operation of complex conjugation; and  $(\Theta_{[j]})_{h,h'} = (-1)^{j+h} \delta_{h',-h}$  being the Wigner time-reversal operator. Using the properties of 4-spinors with respect to chiral

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<sup>7</sup>If accept the way of deriving the equations satisfied by  $(j,0) \oplus (0,j)$  spinors on the base of the Wigner rules (1a) and (1b) in the instant-form of field theory we should assume that  $m \neq 0$ . Therefore, the framework of the front-form relativistic dynamics [21,22,9] could be more convenient for study of the massless limit. We are going to regard this question in following publications.

$\gamma^5$  transformations the equations (8a,8b) in the momentum representation could also be written:

$$i\hat{p}\Upsilon(p^\mu) - m\mathcal{B}(p^\mu) = 0 \quad , \quad (11a)$$

$$i\hat{p}\mathcal{B}(p^\mu) + m\Upsilon(p^\mu) = 0 \quad . \quad (11b)$$

The bispinors (9a-9d) obey the normalization conditions:

$$\overline{\Upsilon}_h(p^\mu)\mathcal{B}_{h'}(p^\mu) = -\overline{\mathcal{B}}_h(p^\mu)\Upsilon_{h'}(p^\mu) = im\delta_{hh'} \quad , \quad (12a)$$

$$\overline{\Upsilon}_h(p^\mu)\Upsilon_{h'}(p^\mu) = \overline{\mathcal{B}}_h(p^\mu)\mathcal{B}_{h'}(p^\mu) = 0 \quad , \quad (12b)$$

$$\Upsilon_h^\dagger(p^\mu)\Upsilon_{h'}(p^\mu) = \mathcal{B}_h^\dagger(p^\mu)\mathcal{B}_{h'}(p^\mu) = p_0\delta_{hh'} \quad , \quad (12c)$$

$$\Upsilon_h^\dagger(p^\mu)\mathcal{B}_{h'}(p^\mu) = \mathcal{B}_h^\dagger(p^\mu)\Upsilon_{h'}(p^\mu) = \begin{cases} +p_3 & , \quad \text{if } h = +, h' = + \\ -p_3 & , \quad \text{if } h = -, h' = - \\ p_l & , \quad \text{if } h = +, h' = - \\ p_r & , \quad \text{if } h = -, h' = + \end{cases} \quad (12d)$$

The properties of 4-spinors of the second kind under space inversion are different from the 4-spinors of the first kind:<sup>8</sup>

$$S_{[1/2]}^s\Upsilon_h(p'^\mu) = -i\mathcal{B}_h(p^\mu) \quad , \quad (13a)$$

$$S_{[1/2]}^s\mathcal{B}_h(p'^\mu) = +i\Upsilon_h(p^\mu) \quad . \quad (13b)$$

In the coordinate representation the obtained equations (8a,8b) yield<sup>9</sup>

$$\left[\gamma^5\gamma^\mu\partial_\mu + m\right]\Psi(x) = 0 \quad , \quad (16)$$

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<sup>8</sup>Of course, the reader is right to ask the question: what could we obtain if define the parity operator according to the anomalous representation of the inversion group, ref. [6,5]? Within the framework of this essay we are still going to accept the viewpoint of Prof. Ahluwalia, ref. [10]: the operator of parity, charge conjugation and time reversal do not depend on a specific wave equation,  $S_{[1/2]}^s = e^{i\theta_{[1/2]}^s}\gamma_0$ . “Without this being true we would not even know how to define charge self/anti-self conjugate  $(j,0) \oplus (0,j)$  spinors.”

<sup>9</sup> Taking into account the unusual properties of the 4-spinors  $\Upsilon_\pm$  and  $\mathcal{B}_\pm$  with respect to the parity conjugation (13a,13b) we conclude that at the classical level Eq. (16) can be put in the form  $(x' = (x_0, -\mathbf{x}), \wp_{u,v} = \pm 1)$

$$i\hat{\partial}_x\Psi(x) - \wp_{u,v}m\gamma_0\Psi(x') = 0 \quad , \quad (14)$$

or

$$i\hat{\partial}_{x'}\Psi(x') - \wp_{u,v}m\gamma_0\Psi(x) = 0 \quad . \quad (15)$$

Investigations of physical consequences following from these equations we leave for future publications. An interesting paper in this direction is ref. [23].

provided that  $\Upsilon_h(p^\mu)$  are regarded as positive-energy solutions and  $\mathcal{B}_h(p^\mu)$ , as negative-energy solutions.<sup>10</sup> By means of simple calculations one can derive the adjoint equation:

$$\bar{\Psi}(x) \left[ \gamma^5 \gamma^\mu \overleftarrow{\partial}_\mu + m \right] = 0 \quad . \quad (17)$$

The equation (16) has been discussed in the old literature [5] and, recently, has been derived [18,19] from the consideration of the Majorana-McLennan-Case construct [24,25] with self/anti-self charge conjugate spinors, developed by Ahluwalia [9,10]. The equation is connected by unitary transformation  $\mathbf{U} = (1 - i\gamma_5)/\sqrt{2}$  with the usual Dirac equation:<sup>11</sup>

$$\begin{aligned} [i\gamma^\mu \partial_\mu - m] \psi(x) &= 0 \quad \rightarrow \\ \rightarrow [i\mathbf{U}^{-1} \gamma^\mu \mathbf{U} \partial_\mu - m] (\mathbf{U}^{-1} \psi(x)) &= 0 \quad \rightarrow \quad [\gamma^5 \gamma^\mu \partial_\mu + m] \Psi(x) = 0 \quad . \end{aligned} \quad (18)$$

According to the modern literature nothing would be changed. Let us still assume that Prof. Dirac were found the equation (16), but not Eq. (18). What mathematical framework were we have now?

*Lagrangian.*

The first difference from the Dirac theory arises when we try to construct the Lagrangian.

**Theorem:** There does not exist the Lagrangian in terms of independent field variables  $\Psi$  and  $\bar{\Psi}$ , that could lead to the Lagrange-Euler equations of the form (16) and (17).

**Proof:**<sup>12</sup> Most general form of the Lagrangian in field variables  $\Psi$  and  $\bar{\Psi}$ ,  $\partial_\mu \Psi$  and  $\partial_\mu \bar{\Psi}$ , that leads to the Lagrange-Euler equations of the first order, is

$$\begin{aligned} \mathcal{L} = & a_1 \bar{\Psi} \gamma^\mu \partial_\mu \Psi + a_2 \bar{\Psi} \gamma^\mu \gamma^5 \partial_\mu \Psi + a_3 \partial_\mu \bar{\Psi} \gamma^\mu \Psi + a_4 \partial_\mu \bar{\Psi} \gamma^\mu \gamma^5 \Psi + \\ & + a_5 \bar{\Psi} \Psi + a_6 \bar{\Psi} \gamma^5 \Psi + a_7 \bar{\Psi} \sigma^{\mu\nu} \partial_\nu \Psi + a_8 \partial_\nu \bar{\Psi} \sigma^{\mu\nu} \Psi \quad . \end{aligned} \quad (19)$$

Therefore,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \Psi} &= a_3 \partial_\mu \bar{\Psi} \gamma^\mu + a_4 \partial_\mu \bar{\Psi} \gamma^\mu \gamma^5 + a_5 \bar{\Psi} + a_6 \bar{\Psi} \gamma^5 + a_8 \partial_\nu \bar{\Psi} \sigma^{\mu\nu} \quad , \\ \frac{\partial \mathcal{L}}{\partial \bar{\Psi}} &= a_1 \gamma^\mu \partial_\mu \Psi + a_2 \gamma^\mu \gamma^5 \partial_\mu \Psi + a_5 \Psi + a_6 \gamma^5 \Psi + a_7 \sigma^{\mu\nu} \partial_\nu \Psi \quad , \end{aligned}$$

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<sup>10</sup>For the moment let us still note that we do not have a strong theoretical principle for the used interpretation of the 4-spinors. The determinants of both Eq. (8a) and Eq. (8b) (from which we find the dispersion relations) provide two signs of the energy  $E = \pm \sqrt{\mathbf{p}^2 + m^2}$ . If assume that  $\Upsilon_h(p^\mu)$  (and  $\mathcal{B}_h(p^\mu)$ ) correspond to the negative (positive) energies we have to use the equation with the opposite sign at the mass term in the coordinate representation. The same situation exists in the usual Dirac equation, refs. [1,2,5,3]. *E. g.*, Prof. M. Markov intended to utilize these two types of the Dirac fields for explanation of mass difference between muon and electron. With an appearance of indications at the third family of leptons (quarks) this idea has been forgotten.

<sup>11</sup> For the sake of completeness let us note that a Pauli term, that could describe interactions of the particle possessing an anomalous magnetic moment, is invariant with respect to the transformation:  $\mathbf{U}^{-1} \sigma^{\mu\nu} \mathbf{U} = \sigma^{\mu\nu}$ .

<sup>12</sup> Cf. with a consideration of vector Lagrangians in ref. [26].

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)} &= a_1 \bar{\Psi} \gamma^\mu + a_2 \bar{\Psi} \gamma^\mu \gamma^5 + a_{7\nu} \bar{\Psi} \sigma^{\nu\mu} \quad , \\ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\Psi})} &= a_3 \gamma^\mu \Psi + a_4 \gamma^\mu \gamma^5 \Psi + a_{8\nu} \sigma^{\nu\mu} \Psi \quad .\end{aligned}$$

From the corresponding Lagrange-Euler equation for  $\Psi$  one has  $a_4 - a_2 = 1$  in order to obtain (16); from the adjoint equation,  $a_2 - a_4 = 1$ . Simultaneous satisfaction of these equations is impossible. Theorem was proven.

How should we manage? Following to the logic of the papers [1,2,5,7,8] I propose to introduce the Dirac “double”, another field that satisfies the equation:

$$\left[ \gamma^5 \gamma^\mu \partial_\mu - m \right] \Psi_2(x) = 0 \quad , \quad (20)$$

and its adjoint

$$\bar{\Psi}_2(x) \left[ \gamma^5 \gamma^\mu \overleftarrow{\partial}_\mu - m \right] = 0 \quad . \quad (21)$$

By using the concept of the two Dirac doubles one can define, *e.g.*, the following Lagrangian:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2} \left[ \bar{\Psi}_2 \gamma^\mu \gamma^5 \partial_\mu \Psi_1 + \partial_\mu \bar{\Psi}_1 \gamma^\mu \gamma^5 \Psi_2 - \bar{\Psi}_1 \gamma^\mu \gamma^5 \partial_\mu \Psi_2 - \partial_\mu \bar{\Psi}_2 \gamma^\mu \gamma^5 \Psi_1 \right] - \\ &\quad - m \left[ \bar{\Psi}_1 \Psi_2 + \bar{\Psi}_2 \Psi_1 \right] \quad ,\end{aligned} \quad (22)$$

which is Hermitian. It leads to the required equations (16,17,20,21). Of course, the Lagrangian is defined within an accuracy of the overall arbitrary constant term (*cf.* Eq. (16) of ref. [3b]). In this paper we choose it equal to the unit.

*Relativistic covariance.*

In order the equation (for  $\Psi_1$  or  $\Psi_2$ ) to be covariant it is necessary that under Lorentz transformation  $x'^\mu = L^\mu_\nu x^\nu$  and  $\Psi'(x') = \mathbf{S}(L)\Psi(x)$  the primed wave function satisfies the same equation:

$$\left[ \gamma^5 \gamma^\mu \partial'_\mu + m \right] \Psi'(x') = 0 \quad (23)$$

or

$$\mathbf{S}^{-1}(L) \left[ \gamma^5 \gamma^\mu \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial}{\partial x^\nu} + m \right] \mathbf{S}(L) \Psi(x) = 0 \quad . \quad (24)$$

Therefore, we have

$$\mathbf{S}^{-1}(L) \gamma^5 \gamma^\alpha \mathbf{S}(L) = L^\alpha_\nu \gamma^5 \gamma^\nu \quad . \quad (25)$$

Restricting ourselves by an infinitesimal proper transformation which may be written as

$$L^\mu_\nu = g^\mu_\nu + \omega^\mu_\nu \quad , \quad (26)$$

where the infinitesimal matrix  $\omega_{\mu\nu}$  is antisymmetric, one can obtain the same result that for the Dirac theory. The generators of the Lorentz transformations are

$$N_{\Psi\Psi}^{\mu\nu} = -\frac{i}{4}\sigma^{\mu\nu} \quad , \quad N_{\overline{\Psi}\overline{\Psi}}^{\mu\nu} = +\frac{i}{4}\sigma^{\mu\nu} \quad . \quad (27)$$

Let us still not forget about the possibility of combining the Lorentz and chiral transformations, ref. [8]. It is the case when the equation (16) comes over to Eq. (20), and reverse. The transformed set of equations leaves to be unchanged. In this case the matrix of the transformation would be  $\mathbf{S}(L) = \gamma^5 \exp(-i\sigma^{\mu\nu}\omega_{\mu\nu}/4)$  and the generators are

$$N_{\Psi_1\Psi_2}^{\mu\nu} = -\frac{i}{4}\gamma^5\sigma^{\mu\nu} \quad , \quad N_{\overline{\Psi}_1\overline{\Psi}_2}^{\mu\nu} = +\frac{i}{4}\gamma^5\sigma^{\mu\nu} \quad . \quad (28)$$

*Dynamical invariants.*

By means of the standard procedure [27,28] from the Lagrangian (22) one can obtain dynamical invariants (that are deduced as a consequence of the uniformity, the isotropy of space-time and of the invariance of the Lagrangian with respect to gradient transformations of the first kind):

- The current vector and the charge operator:

$$\begin{aligned} J^\mu &= i \sum_i \left[ \overline{\Psi}_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \overline{\Psi}_i)} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi_i)} \Psi_i \right] = \\ &= i \left[ \overline{\Psi}_1 \gamma^\mu \gamma^5 \Psi_2 - \overline{\Psi}_2 \gamma^\mu \gamma^5 \Psi_1 \right] \quad , \end{aligned} \quad (29)$$

$$Q = \int d^3x J^0(x) \quad . \quad (30)$$

- The energy-momentum tensor and the 4-vector of energy-momentum:

$$\begin{aligned} \Theta^{\mu\nu} &= \sum_i \partial^\nu \overline{\Psi}_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \overline{\Psi}_i)} + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi_i)} \partial^\nu \Psi_i - g^{\mu\nu} \mathcal{L} = \\ &= \frac{1}{2} \left[ \partial^\nu \overline{\Psi}_1 \gamma^\mu \gamma^5 \Psi_2 + \overline{\Psi}_2 \gamma^\mu \gamma^5 \partial^\nu \Psi_1 - \partial^\nu \overline{\Psi}_2 \gamma^\mu \gamma^5 \Psi_1 - \overline{\Psi}_1 \gamma^\mu \gamma^5 \partial^\nu \Psi_2 \right] - \mathcal{L} g^{\mu\nu} \quad , \end{aligned} \quad (31)$$

$$P_\mu = \int d^3x \Theta_\mu^0(x) \quad . \quad (32)$$

- The angular-momentum tensor and the spin operator:

$$\begin{aligned} J^{\alpha,\mu\nu} &= x^\mu \Theta^{\alpha\nu} - x^\nu \Theta^{\alpha\mu} + 2 \sum_{ij} \left[ \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \Psi_i)} N_{ij}^{\mu\nu} \Psi_j + \overline{\Psi}_i N_{ij}^{\mu\nu} \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \overline{\Psi}_j)} \right] = \\ &= x^\mu \Theta^{\alpha\nu} - x^\nu \Theta^{\alpha\mu} + \frac{i}{4} \left\{ \overline{\Psi}_1 \left[ \gamma^\alpha \gamma^5 \sigma^{\mu\nu} + \sigma^{\mu\nu} \gamma^\alpha \gamma^5 \right] \Psi_2 - \overline{\Psi}_2 \left[ \gamma^\alpha \gamma^5 \sigma^{\mu\nu} + \sigma^{\mu\nu} \gamma^\alpha \gamma^5 \right] \Psi_1 + \right. \\ &\quad \left. + \overline{\Psi}_1 \left[ \gamma^\alpha \sigma^{\mu\nu} + \sigma^{\mu\nu} \gamma^\alpha \right] \Psi_1 - \overline{\Psi}_2 \left[ \gamma^\alpha \sigma^{\mu\nu} + \sigma^{\mu\nu} \gamma^\alpha \right] \Psi_2 \right\} \quad , \end{aligned} \quad (33)$$

$$J^{\mu\nu} = \int d^3x J^{0,\mu\nu}(x) \quad , \quad (\hat{W} \cdot n) = S^{12} \quad , \quad \text{if } \mathbf{n} \parallel \mathbf{p} \parallel OZ \quad (34)$$



( $S^{12}$  denotes the spin part of the angular momentum operator;  $\hat{W}_\mu$  is the Pauli-Lyuban'sky operator).

By using the plane-wave expansion:

$$\Psi_1(x) = \sum_h \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2p_0} \left[ \Upsilon_h^{(1)}(p^\mu) a_h(p^\mu) e^{-ip \cdot x} + \mathcal{B}_h^{(1)}(p^\mu) b_h^\dagger(p^\mu) e^{ip \cdot x} \right] , \quad (35a)$$

$$\Psi_2(x) = \sum_h \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2p_0} \left[ \Upsilon_h^{(2)}(p^\mu) c_h(p^\mu) e^{-ip \cdot x} + \mathcal{B}_h^{(2)}(p^\mu) d_h^\dagger(p^\mu) e^{ip \cdot x} \right] \quad (35b)$$

in the Fock space we obtain the following results:

$$\hat{Q} = \frac{1}{2} \sum_h \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2p_0} \left[ a_h^\dagger(p^\mu) c_h(p^\mu) + c_h^\dagger(p^\mu) a_h(p^\mu) - b_h(p^\mu) d_h^\dagger(p^\mu) - d_h(p^\mu) b_h^\dagger(p^\mu) \right] , \quad (36)$$

$$\hat{\mathcal{H}} = \hat{P}_0 = \frac{1}{2} \sum_h \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2p_0} p_0 \left[ a_h^\dagger(p^\mu) c_h(p^\mu) + c_h^\dagger(p^\mu) a_h(p^\mu) + b_h(p^\mu) d_h^\dagger(p^\mu) + d_h(p^\mu) b_h^\dagger(p^\mu) \right] , \quad (37)$$

and

$$\begin{aligned} (\hat{W} \cdot n) &= \frac{1}{4} \sum_{hh'} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2p_0} \xi_h(\boldsymbol{\sigma} \cdot \mathbf{n}) \xi_{h'} \times \\ &\times \left[ a_h^\dagger(p^\mu) c_{h'}(p^\mu) + c_{h'}^\dagger(p^\mu) a_h(p^\mu) - b_h(p^\mu) d_{h'}^\dagger(p^\mu) - d_{h'}(p^\mu) b_h^\dagger(p^\mu) + \right. \\ &\left. + i a_h^\dagger(p^\mu) a_{h'}(p^\mu) - i c_{h'}^\dagger(p^\mu) c_h(p^\mu) + i b_h(p^\mu) b_{h'}^\dagger(p^\mu) - i d_{h'}(p^\mu) d_h^\dagger(p^\mu) \right] . \end{aligned} \quad (38)$$

Like the usual formulation [28, p.145] we have to subtract the vacuum contributions in these dynamical invariants. Next, let me note an interesting feature of this formulation. For the first sight we obtained the positive-definite Hamiltonian. Does this fact signify that there are no reasons for introduction of anticommutation relations between operators  $a_h(p^\mu)$  and  $c_h^\dagger(p^\mu)$  (as well as between  $b_h(p^\mu)$  and  $d_h^\dagger(p^\mu)$ ) ?

*Propagators.*

Using the known procedure of Stückelberg and Feynman for the field  $\Psi_1$  (or  $\Psi_2$ ) one can obtain a local propagator, but it is not the Green's function of the corresponding equation:

$$\begin{aligned} &\left[ \gamma^5 \gamma^\mu \partial_\mu^{x_2} + m \right] \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2p_0} \left[ a \theta(t_2 - t_1) \sum_h \Upsilon_h(p^\mu) \otimes \bar{\Upsilon}_h(p^\mu) e^{-ip \cdot (x_2 - x_1)} + \right. \\ &\left. + b \theta(t_1 - t_2) \sum_h \mathcal{B}_h(p^\mu) \otimes \bar{\mathcal{B}}_h(p^\mu) e^{ip \cdot (x_2 - x_1)} \right] = \frac{a}{2} \gamma^5 \delta^{(4)}(x_2 - x_1) , \end{aligned} \quad (39)$$

provided that  $a = -b$ . The same result is obtained for the second equation (20) if we use the 4-spinors satisfying the second equation (20)<sup>13</sup>

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<sup>13</sup>The spinorial basis for 4-spinors satisfying the second equation is chosen as

$$\Upsilon_h^{(2)}(p^\mu) = -i\gamma^5 \Upsilon_h^{(1)}(p^\mu) \quad , \quad \mathcal{B}_h^{(2)}(p^\mu) = i\gamma^5 \mathcal{B}_h^{(1)}(p^\mu) \quad . \quad (41a)$$

$$\bar{\Upsilon}_h^{(2)}(p^\mu) = -i\bar{\Upsilon}_h^{(1)}(p^\mu)\gamma^5 \quad , \quad \bar{\mathcal{B}}_h^{(2)}(p^\mu) = i\bar{\mathcal{B}}_h^{(1)}(p^\mu)\gamma^5 \quad . \quad (41b)$$

We have used above that

$$\sum_h \Upsilon_h^{(1)}(p^\mu) \otimes \bar{\Upsilon}_h^{(1)}(p^\mu) = \sum_h \mathcal{B}_h^{(2)}(p^\mu) \otimes \bar{\mathcal{B}}_h^{(2)}(p^\mu) = \frac{1}{2} [\hat{p} + im\gamma^5] \quad , \quad (42a)$$

$$\sum_h \Upsilon_h^{(2)}(p^\mu) \otimes \bar{\Upsilon}_h^{(2)}(p^\mu) = \sum_h \mathcal{B}_h^{(1)}(p^\mu) \otimes \bar{\mathcal{B}}_h^{(1)}(p^\mu) = \frac{1}{2} [\hat{p} - im\gamma^5] \quad . \quad (42b)$$

Nevertheless, taking into account that

$$\sum_h \Upsilon_h^{(1)}(p^\mu) \otimes \bar{\mathcal{B}}_h^{(1)}(p^\mu) = - \sum_h \mathcal{B}_h^{(2)}(p^\mu) \otimes \bar{\Upsilon}_h^{(2)}(p^\mu) = \frac{1}{2} [\gamma^5 \hat{p} - im] \quad , \quad (43a)$$

$$\sum_h \mathcal{B}_h^{(1)}(p^\mu) \otimes \bar{\Upsilon}_h^{(1)}(p^\mu) = - \sum_h \Upsilon_h^{(2)}(p^\mu) \otimes \bar{\mathcal{B}}_h^{(2)}(p^\mu) = \frac{1}{2} [\gamma^5 \hat{p} + im] \quad , \quad (43b)$$

we obtain the needed result:

$$\begin{aligned} & \left[ \gamma^5 \gamma^\mu \partial_\mu^{x_2} + m \right] \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{2p_0} \left[ a \theta(t_2 - t_1) \sum_h \Upsilon_h^{(1)}(p^\mu) \otimes \bar{\mathcal{B}}_h^{(1)}(p^\mu) e^{-ip \cdot (x_2 - x_1)} + \right. \\ & \left. + b \theta(t_1 - t_2) \sum_h \mathcal{B}_h^{(1)}(p^\mu) \otimes \bar{\Upsilon}_h^{(1)}(p^\mu) e^{ip \cdot (x_2 - x_1)} \right] = \delta^{(4)}(x_2 - x_1) \quad , \quad \text{if } a = -b = -2 \quad , \quad (44) \end{aligned}$$

and

$$\begin{aligned} & \left[ \gamma^5 \gamma^\mu \partial_\mu^{x_2} - m \right] \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{p_0} \left[ a \theta(t_2 - t_1) \sum_h \Upsilon_h^{(2)}(p^\mu) \otimes \bar{\mathcal{B}}_h^{(2)}(p^\mu) e^{-ip \cdot (x_2 - x_1)} + \right. \\ & \left. + b \theta(t_1 - t_2) \sum_h \mathcal{B}_h^{(2)}(p^\mu) \otimes \bar{\Upsilon}_h^{(2)}(p^\mu) e^{ip \cdot (x_2 - x_1)} \right] = \delta^{(4)}(x_2 - x_1) \quad , \quad \text{if } a = -b = 2 \quad . \quad (45) \end{aligned}$$

Finally,

$$S_F^{(1)}(x) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot x} \frac{i\hat{p}\gamma^5 - m}{p^2 - m^2 + i\epsilon} \quad , \quad (46a)$$

$$S_F^{(2)}(x) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot x} \frac{i\hat{p}\gamma^5 + m}{p^2 - m^2 + i\epsilon} \quad . \quad (46b)$$

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$$\Upsilon_+^{(2)}(\hat{p}^\mu) = \sqrt{\frac{m}{2}} \begin{pmatrix} -i \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad , \quad \Upsilon_-^{(2)}(\hat{p}^\mu) = \sqrt{\frac{m}{2}} \begin{pmatrix} 0 \\ -i \\ 0 \\ 1 \end{pmatrix} \quad , \quad (40a)$$

$$\mathcal{B}_+^{(2)}(\hat{p}^\mu) = \sqrt{\frac{m}{2}} \begin{pmatrix} i \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad , \quad \mathcal{B}_-^{(2)}(\hat{p}^\mu) = \sqrt{\frac{m}{2}} \begin{pmatrix} 0 \\ i \\ 0 \\ 1 \end{pmatrix} \quad . \quad (40b)$$

The role of propagators is similar to the usual Dirac theory: to propagate the positive frequencies toward positive times and the negative ones, backward in time [29].

*Physical contents and concluding remarks.*

First of all, I would like to note that one has a variety of possibilities of physical interpretation of the results obtained in the previous sections. It depends on the connection between creation (annihilation) operators of the two fields and setting the (anti)commutation relations between them.

Next, one has a puzzled problem with the imaginary part of the Pauli-Lyuban'sky operator. It is very natural to regard the particular cases when these imaginary terms are cancelled each other. The creation and annihilation operators of two parts of the doublet  $\Psi_1$  and  $\Psi_2$  are then connected by the one of the following manners:

- $c_h(p^\mu) = +a_h(p^\mu)$ ,  $d_h(p^\mu) = -b_h(p^\mu)$ ; or  $c_h(p^\mu) = -a_h(p^\mu)$ ,  $d_h(p^\mu) = +b_h(p^\mu)$ . From the definitions of the plane-wave expansion and from the relations (41a,41b), between 4-spinors  $\Upsilon^{(1,2)}$  and  $\mathcal{B}^{(1,2)}$  satisfying the first (16) and the second equation (20) one follows that  $\Psi_2(x) = \pm i\gamma^5 \Psi_1(x)$ . If we set anticommutation relations

$$\{a_h(p^\mu), a_{h'}^\dagger(q^\mu)\}_+ = (2\pi)^3 2p_0 \delta^{(3)}(\mathbf{p} - \mathbf{q}) \delta_{hh'} \quad , \quad (47a)$$

$$\{b_h(p^\mu), b_{h'}^\dagger(q^\mu)\}_+ = (2\pi)^3 2p_0 \delta^{(3)}(\mathbf{p} - \mathbf{q}) \delta_{hh'} \quad , \quad (47b)$$

we can assure ourselves that the formalism describes the charged particles of the opposite sign; the antiparticle could be considered in the accordance with the common-used “hole” interpretation. We have the Dirac’s electron and positron (or an muon and an antimuon), indeed.

- $c_h(p^\mu) = a_h(p^\mu)$ ,  $d_h(p^\mu) = b_h(p^\mu)$ . Regarding the dynamical invariants (36-38) we obtain that the charge operator could be interpreted as the particle number operator (there are no particles with the opposite sign of the charge). The energy is not a positive-definite quantity (antiparticles contribute to the energy with the opposite sign). A compatibility of these two conclusions is unclear. Perhaps, this case could be relevant to description of neutral particles. If we were able to set the commutation (*not* anticommutation) relations for this case, the physical interpretation could be completely different.
- $c_h(p^\mu) = \pm i a_h(p^\mu)$ ,  $d_h(p^\mu) = \pm i b_h(p^\mu)$ . Both the charge operator, the Hamiltonian and the Pauli-Lyuban'sky operator are equal to zero. The interpretation of this physical state is completely unclear (there are no particles at all?). Perhaps, one can find some reminiscences with “physical excitations” discussed in ref. [8d] and [30,31].

I would also like to draw reader’s attention to the problem of the choice of field variables to use for the variation. It is easy to see that, *e.g.*, under the substitution  $\Psi_2 \rightarrow i\Psi_2$  the Lagrangian (22) is changed and it leads to the different expressions for dynamical invariants<sup>14</sup>.

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<sup>14</sup>Of course, the another expressions for dynamical invariants are connected with the first ones by certain transformations of creation (annihilation) operators.

Next, if start from the usual Dirac Lagrangian, but vary using another field variables (*e.g.*,  $\psi$  and  $\gamma^5\psi$  (or  $\psi$  and  $(1/\sqrt{2})(1 - i\gamma^5)\psi$ ) one can obtain physical excitations of the very different nature. Could these physical excitations, following from the Dirac Lagrangian, be relevant to describing neutrino (or, even, intermediate vector bosons)? Could we observe transitions between the Dirac charged states described by Eq. (18) and the states described by equations (16,20) ? If yes, how should we correct the Feynman diagram technique? It is undoubtedly that we would obtain the very different expressions for self-energies, vertex functions and for other diagrams involving the fields  $\Psi_1$  and  $\Psi_2$ . Finally, Ryder [13] writes: “When a particle is at rest, one cannot define its spin as either left- or right-handed, so  $\phi_R(0) = \phi_L(0)$ .” In the papers [7,10] it was paid attention to the more general forms of the Faustov-Ryder-Burgard relation. Here we have considered another form of the relation between left- and right- spinors at rest and, as a result, we have deduced an interesting model based on this principle. What physical excitations could we obtain if set the Faustov-Ryder-Burgard relation, *e.g.*, in the another form:  $\phi_R(\hat{p}^\mu) = (\pm i\sqrt{3}/2 - 1/2)\phi_L(\hat{p}^\mu)$  (*cf.* with the formulas (50) in ref. [10b]) ? Or, even in more general form?<sup>15</sup> Let us also note that the Lagrangian (22) is compatible with a gauge principle. So, the connection of the presented formalism with non-Abelian gauge theories is transparent. Therefore, it could serve as a ground for recreation of the ideas proposed in the old papers [33,34].

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<sup>15</sup>For the discussion of the most general case of the Faustov-Ryder-Burgard relation see ref. [32].

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